THE PROBLEM OF THE MOON'S MOTION

By Ernest W. Brown

The beginning of the problem which is known as that of the motion of the Moon about the Earth, or briefly, the lunar theory, goes back to the earliest time in history, when man began consciously to observe nature. The Sun he would easily consider as part of the regular order of things dividing day from night and so much a part of daily life from his infancy as to escape his curiosity. The Moon, however, with its constant changes in appearance and position and yet with a certain degree of regularity would probably be one of the earliest objects of speculation. It formed the first rough measure of the passage of days to a race whose cradle was sufficiently far from regions where the Sun never rises far above the horizon in winter. To this day it is used by uncivilized tribes to measure off periods, rather than the year, which holds too many alternations of light and dark to be counted.

The earliest record we have comes from the clay tablets of the Babylonians. They knew that there existed a cycle of between eighteen and nineteen years—the Saros—in which the eclipses of the Moon repeated themselves. This fact alone shows careful observation and record which must have extended over at least a century and probably very much longer. It is interesting to speculate on various deductions which might be made from this record alone. Clear skies must have been the rule and cloudy nights rather rare exceptions. For the phenomenon had to be seen and noted as something unusual and not to be explained by clouds. And the observers also must have had the opportunity to see plainly most of those eclipses that took place. Again the knowledge gained must have been handed on to the next generation at least and more probably thru several generations in an age when scientific record was the exception, giving the impression that it was cared for by some body of men who formed a school or priesthood. The knowledge may have been used for impressing the populace by predictions of events which would not need to be explained away. But I think that the very nature of the record seems to indicate that it was not kept as a "secret of magic," at least in the later days. The Babylonians were fond of numbers and used them in many ways; they could "write a washing bill in Babylonic Cuneiform."

A gap of two thousand years in our knowledge of the past brings us to Hipparchus, the Greek, who lived near the golden age of that civilization, and later to Ptolemy, who gathered together the work of his immediate predecessors. At this time they had observed enough to become puzzled at the want of regularity in the Moon's motion amongst the stars. It had not the perfection of the circle which the Greeks felt should belong to the motions of the heavenly bodies and the names of some of the deviations from uniform circular motion date from this period. Knowledge grew slowly in the next 1500 years. It was not until the time of Tycho Brahe, the observer who recorded the motions of the planets and Sun and Moon, that the foundations were laid for a more perfect classification of the peculiarities of these motions. Then came Kepler who, abandoning the idea that circles were the only curves which were suitable to represent all the different motions of these bodies, finally after many years of patient work on the observations of Tycho Brahe, hit on the ellipse as the best solution of the problem.

This is the end of the first stage—the observations gathered together, arranged in orderly fashion and so grouped that a simple well-known curve would describe their main features. This was at least true of the motion of the planets round the Sun, once the idea for which Galileo is the historical protagonist had been generally accepted by the scientific men of the time. It was less true of the Moon, which showed deviations which could not be grouped in so simple a manner. Something more was needed not only to explain these, but also to find the reason why all the planets moved in ellipses with the Sun in one focus which was common to all of them. And Kepler had found two other facts which connected their rates of motion, the times in which they described their circuits and their distances from the Sun. The answer was supplied by Isaac Newton, who was able to give a solution—the law of gravitation—which could put in a single statement all the facts which Kepler had brought together and more particularly which showed that the motions of the Moon were no exception.

This second stage reached the heart of the problem. Once the laws which all matter obeys had been fully formulated and tested with such observations as were then available, it was necessary to find out all the consequences of these laws. For this the older methods of mathematics were quite insufficient. It was necessary to find some weapon much more powerful than the simple arith-

metic, algebra and geometry which were the chief content of mathematics as taught in Newton's time. Newton and his contemporary Leibnitz furnished this, the one by the invention of the method of fluxions or rates, as we should now call them, and the other by what amounted then to the same thing expressed in a more suggestive notation, the calculus. From that time to the present day the mathematical astronomer with the assistance of his co-operator in the observatory, has been testing the full consequences of the law of gravitation.

Man is so constituted that he never remains long content to have arrived at the source of the power which is responsible for the working of a large and complicated machine. He may be convinced that he has arrived at the place where he can learn all that is to be known, but he will soon try to attain that knowledge. The law of gravitation may be responsible for all the working of the solar system and of the stellar universe, but it is only natural to follow out the various lines which radiate from the central power-house and examine in detail each of them to its furthest extent. He will be on the lookout, too, for indications as to other sources of power. To do this in the case of gravitation means careful exploration of every detail, the more careful the more difficult it appears to be to find other sources. And with this law it is peculiarly necessary. It leaves so few things within its province which cannot be traced back to it that the most thoro investigation is necessary if we are to discover anything new. It is here that the Moon comes to play an important part. It is so near to us and gives so many opportunities for thoro observation that we may hope to find out more from it than from any other single body in the heavens. It is true that it is only one of them but it is that one in which the play of the gravitational forces has the most varied effects. Hence it is not strange that more labor has been devoted to elucidating its motions than those of any other body.

What is it that has to be done? The consequences of the law have to be deduced and to be expressed in a form in which they may be most easily compared with observation. This expresses in a single sentence the work to which several of my predecessors have devoted a large part of their energies. I shall try in as few words as possible to give some idea of what it involves. Only one man can be said to have accomplished the whole of it with any degree of completeness. Hansen, who lived in the first half of the nineteenth century,

not only constructed a new method for working out the consequences of the law as applied to the Moon but constructed and calculated tables for simplifying the results so that the labor of finding the position of the Moon at any time should not be too great. Most of the other workers had a less ambitious program or, like Delaunay, died before it was completed. In my own case, there was the foundation of a new method, that given by G. W. Hill, in existence, and the ideas of Hansen and his successors as to how tables should be constructed were available. Of course all the algebraical and numerical calculation had to be done afresh and much greater accuracy and completeness were necessary if the work was to be worth doing at all. Much of it called for careful organization and thoro tests rather than mathematical ability and its completion at the present time is largely due to favorable working conditions and financial assistance forthcoming when it was most needed to provide for the more mechanical parts of the calculations.

What is known as the "theory" consists in constructing a set of differential equations which are the symbolic expression of the laws of motion and the law of gravitation, and then in solving them so as to get the three co-ordinates—the longitude, the latitude and the parallax—expressed in terms of a single variable quantity, the time, so that by substituting any given date in the expressions we may obtain these co-ordinates by straightforward numerical calculation. The problem would be simple enough if a spherical Earth and a spherical Moon were the only bodies to be considered. But the Sun altho so far distant from the Moon compared with the Earth has a much greater mass and its attraction causes considerable deviations from simple elliptic motion. Then the planets *Mercury*, *Venus*, *Mars*, *Jupiter* and *Saturn* all produce sensible effects and we have, too, to take account of the fact that neither the Earth nor the Moon are perfect spheres. The chief difficulty comes from the fact that we can only express the results as sums of many hundreds of terms each of which has portions which come from many different parts of the calculations. I can perhaps best give an idea of the extent by mentioning that the number of figures written mounted to four or five millions not counting algebraic symbols or the figures which pass thru the mind while doing the calculations. This work took nearly 10,000 hours of my own time and that of one computer. In the final results about 1500 terms were left which seemed large enough to be recorded as having an effect to be included when

obtaining the position of the Moon at any time. Practically every term is of the form $a \sin (bt + c)$ that is a harmonic function of the time, in which a, b and c have numerial values which differ from term to term.

The results are then ready for the observer whenever he wishes to compare the theoretical position of the Moon with its observed place. But if he had to calculate from these 1500 terms every time an observation was made the work would be so great that few would be found to undertake it. We can easily estimate what he would have to do. He would have to add 1500 numbers together in three groups and I doubt whether he could get any one of them with less than five minutes' work. The whole would take not less than 100 hours of work and would probably demand double that time. The *American Ephemeris* gives the place of the Moon every hour thruout the year. On this plan some thirteen million numbers would have to be added and the time demanded to get the positions of the Moon for a single year would be at least a million hours. It is to abbreviate this work that Tables of the Moon are constructed.

This is done mainly by two principles. First the fact that the motion is continuous and follows a regular law for a short time enables us to perform the calculation once every twelve hours instead of once an hour. This law tells us that even the shortest of the cycles described by the various terms in the motion is much longer than twelve hours and that if we know the place at 12-hour intervals a simple calculation from the places themselves will give the place at every hour: the process is known as interpolation. The amount of labor involved in this process will be easily understood by the astronomer when the fact is mentioned that differences up to the fourth inclusive are used. The number of items is thus reduced to a million or so.

The second principle is to do all the calculation which can serve for any date before applying the results to a given date. This requires a careful examination of the results in order to see what can to done. First the angle under each sine is tabulated so that its values are half a day apart and the sine of each is multiplied by the coefficient. The sine has the property of repeating its values whenever we add 360° to the angle so that all the values of the term which will ever be needed can be obtained from such a table by interpolation; we can of course find as many intermediate values to save the latter process. This reduces the time for the calculation

of any term from five minutes to perhaps half a minute: the number of hours required is reduced to ten thousand. Next it is found that many of such tables can be added together so as to form only one table. As a matter of fact the 1500 terms in my theory have been placed in fewer than 150 tables, so that the time required is again divided by ten. Finally the translating of the date into the angle is done by a set of tables and simple rules which save more time. It turns out that the ordinary ephemeris computer should be able to do the whole work on the ephemeris for a year in from six to nine months according to the amount of time he has to spend in checking up his work. The whole idea is similar in principle to that which a factory adopts in sending out its product. It ships in bulk and retains bulk as long as possible, breaking into smaller portions at certain central points, in order to distribute to the dealers who finally get out the single packages for sale to the public. The advantage of this method over that of sending each package direct to the consumer needs no comment. If we try to continue the simile further back, the "theory" will compare with the work done in the factory where the raw material furnished by the laws of motion and gravitation is worked up into the finished product, ready for distribution.

I have not mentioned one feature of the theory which is a very important part of the work. The laws do not give certain constants which can only be found from observation. We must, for example, know the average time of revolution, the mean distance and so on by direct observation of the Moon. The process consists in calculating the position with rough values of these constants and correcting them from the differences between theory and observation. If we had to start at the beginning knowing little about these constants the work would be intensely laborious. But perfection of theory and observation have gone hand in hand and we have now got very accurate values of these constants. But still each time the theory is improved the constants need correction. Hence while the Tables were being calculated, the past observations of the Moon were compared with the new theory. Some 20,000 of the Greenwich observations made in the last 150 years were used since the more observations which could be incorporated, the more accurate would be the averages. My own part in this was not a very serious undertaking for all the really laborious work had been done by my predecessors and this had only to be corrected

and discussed afresh. I had the advantage not only of being able to use this work of Airy and Cowell, but also that of Newcomb, who obtained his results from occultations of the stars by the Moon—a quite separate set of observations. The final agreement after the two sets had been corrected and discussed showed that there was little to be desired in the way of accuracy in the constants of the Moon's orbit.

Navigators have almost ceased to use the Moon to obtain their positions at sea, owing to the excellence of modern chronometers and the development of wireless telegraphy, and there are few, if any, other practical needs for accurate observations of the Moon. The objects of this work are therefore chiefly scientific and I shall now try to explain why such a detailed study of the motions of one body of our solar system is worth while. Some mention has been made of the continued interest for itself alone of the workings of the law of gravitation and of the many possibilities which exist in the lunar problem. But at the present time the matter of chief interest is the remainder after the effects of gravitation have all been subtracted. Many of the older sciences have reached the stage in which we can only hope to learn something new by the most careful study and comparison. The lunar theory is more than 230 years old, and during its life has never failed to have many devotees. The last refinements are now necessary. It is for this reason that the attempt was made to leave no gravitational source untouched which could have any effect sensible to observation now and for many years to come. The contest between theory and observation, like that between the shell and armor plate, is continuous. As far as gravitation goes the theory is now in the ascendant and to learn more about the residuals, whose laws of change are as yet uncertain, we need many years of observations as refined as possible.

This seems to be an opportunity to sum up briefly what is now known about the most famous of these residuals. I shall not confuse the question by going into its history. The gradual apparent decrease in the length of the month—usually known as the secular acceleration of the Moon's mean motion can be determined by theory with great exactness. This effect causes the Moon to be ahead of its place six seconds of arc in a century more than if it did not exist. But the effect varies as the square of the time so that 2000 years ago, the accumulated change would

be 2400″ or two-thirds of a degree. It will be seen that we can only test it after very long intervals of time. Now an eclipse is a very sensitive test for such a change. Even a third of this amount will shift the line on which the eclipse is visible so far that the Sun may not appear totally eclipsed at all. Now we have records of the occurrence of eclipses going far back and they indicate a greater change than 6″ in a century. Unfortunately these early records are vague both as to place and time and one is sometimes even in doubt as to whether the eclipse took place at all. Nevertheless taken all together, they give a fair probability that the increase is about 8″ instead of 6″ so that about 2″ is unaccounted for by gravitation.

Cowell and Fotheringham in England have gone a step farther. They have shown that there are two accelerations in question and that we cannot treat them as if the Moon alone were concerned. The 2″ already mentioned is observed as a difference between the places of the Sun and Moon. There is another of nearly the same amount remaining between the place of the Sun and that of the node of the Moon's orbit. Fotheringham has brought additional evidence to bear on the latter by working up some ancient records of occultations. We thus have two angles made by three lines to consider.

Before the latter difference was discovered the former was usually attributed to a slowing down of the Earth's rate of rotation by tidal friction which would produce the same apparent effect. This rate of rotation is in fact the astronomer's clock and at present the most accurate method of testing the rate of the clock over long periods of time is by means of the Moon. But this comparison is only useful if we know that one of the two is going perfectly. We could in fact attribute the difference either to a speeding up of the Moon's motion or to a slowing down of the Earth's rotation. The latter has what seemed to be a reasonable explanation in the friction caused by the tides. But the second difference probably cannot be attributed to such a cause and we must either abondon the hypothesis or find some other reason for the second difference. Some light may probably be thrown on the question by calculations as to the amount of this friction which have been made lately but which are not yet in a state to use as an argument. There is, I think, a growing conviction that the Earth's average rate of rotation has not sensibly changed

within historic times. Cowell suggested that both differences could be explained by a change in the length of the year, but here we are little better off because we know of no reason why this should alter.

Even more puzzling are the curious semi-regular fluctuations of the Moon from its theoretical position which were first discovered by Simon Newcomb. Some six years ago I discussed these in Australia and showed that the Earth and Mercury exhibited deviations of a similar character tho smaller in amount. A little later Glauert confirmed these results and added *Venus* to the group. Following Larmor he suggested that they might be explained by a fluctuation in the rate of rotation of the Earth. But here again causes requisite to produce the amount required seem to be absent. The irregular character and comparatively great magnitude of these fluctuations suggests that there is some set of forces acting on the bodies of the solar system which are related to the known irregular changes in the condition of the Sun. Hypotheses concerning the magnetic and electrical condition of the Sun, Earth and the other bodies are easy to postulate but difficult to confirm. Nevertheless evidence of some connection between these and the Moon's motion are not altogether lacking. The period of rotation of the Sun's magnetic axes found at Mount Wilson is sufficiently close to that of one of the principal lunar periodic terms known as the evection to suggest a long-period change by the phenomenon known as resonance. Long continued observation will be necessary to enable us to see whether the suggestion is of value. What is already known and being discovered, is, however, a sufficient basis to keep the suggestion in mind.

One naturally asks whether the new theories concerning the ultimate structure and nature of matter can have any bearing on these questions. The new law of gravitation, or perhaps stated more correctly, the correction to the Newtonian law of gravitation suggested by Einstein in connection with his relativity theory, is a natural suggestion. One can say almost at once from the form of the law that it is powerless to explain any of these observations of the Moon. But de Sitter has worked the results out with some care and he finds no effect of this kind. There is a slight effect of another kind, but it is too small to be tested in the present state of the theory and the observations. Indeed of the rather numerous

anomalies in the motions of the bodies of the solar system it only accounts for one, namely, the motion of the perihelion of *Mercury*, and the strength of the evidence rests solely on the fact that the numerical result is obtained without the introduction of a new constant. This new theory of Einstein's is perhaps the most remarkable working hypothesis of modern times, the triumph of the mathematician and the despair of the physicist, but one must regard it at present as simply a working hypothesis with only a single positive test yet applied as far as the gravitational part of it is concerned.

Some by-products of the work are worth a notice. One is the rate of change of the angle between the ecliptic and the equator. Newcomb believed that the value he got from the occultations was superior to that which could be obtained from observations of the Sun, and as my value agrees with his and has a smaller probable error, I should be disposed to think that, with the great weight of his authority on such questions, the latest value is probably the most accurate of any so far found. A second is the ellipticity of the Earth's figure. Here there is a small but sensible difference of about one per cent from that obtained by geodetic measures and by pendulum observations of gravity. While there are possibilities that the value obtained from the Moon's motion may be made to agree with that obtained from terrestrial measurements by altering certain other rather doubtful constants, I think that the weight of evidence is against such an explanation.

I have so far not mentioned methods by which the place of the Moon is observed. The method of recording the passage of the Moon across the meridian has been used for many years and furnishes by far the largest number of recorded observations which we possess. Within the past few years Greenwich has also added extra-meridian observations with the alt-azimuth and has also used the crater Mösting A as a fiducial point on the Moon's surface as well as the edge of the limb. Occultations of stars are nearly always recorded and they have the great advantage that the instrumental equipment can be of a much less refined nature than that required for meridian observations. Some ten years ago I discussed the question of new methods, using photography with Professor Pickering, which had the fortunate outcome, a few month's later, of producing positions of the Moon more accurate

than any of the past methods[1]. Professor Russell, at Princeton, has superintended the measurement and reduction of all the Harvard plates. The latter is somewhat laborious but the results show a smaller probable error than the Greenwich observations and this probable error is mainly due to inaccuracies in the star-positions of the catalogs. Thus with improvements of the latter we should be able greatly to improve the observations of the Moon. There are, it is true, difficulties connected with the illumination of the limb, but these do not seem to be insuperable. The fact that the probable error depends chiefly on those of the star-positions naturally suggests that the Moon might possibly be used to obtain differences of the right ascension of stars which are too far apart to be measured on a single plate. The theoretical motion of the Moon for a short time is, I believe, sufficiently accurate for this purpose. I may add that the chief object for which the method was suggested, namely, to test for systematic error in the Greenwich observations, was attained; no such error was found.

Thus perfection in knowledge of the Moon's motion leads to applications in several directions. It leads not only to more knowledge of the law of gravitation but furnishes possibilities for use as a clock, for use in comparing positions in the heavens and for use in measurements of the Earth.

The past history of lunar theory has been largely the work of individuals, but in the future it will probably not be so. What we are beginning to recognize more and more is that discovery is becoming less dependent on the work of the individual and passing to that of the group. There will be as great need as ever for the directing hand, but the director becomes less and less the dominating genius. He is rather the dominating personality who can stimulate and inspire those who are working with him, who has the conscious or unconscious knowledge which can cause the ideas which are presented to germinate and grow. There is perhaps no one who can feel the need of the group more keenly than he who has worked on a problem alone for much of his life and who has had little opportunity for intercommunication with the few others in his own line except the meager stimulus afforded by a few printed pages appearing at rare intervals. This group tendency is manifested at the present time in the great industrial

[1]See the article by Professor Russell on another page of this number.—Editor's note.

laboratories, the great institutions for research, the government bureaus, and the universities, where men and women gather together for a common purpose. They are by no means without flaws; every human institution has many defects where the ambitions of the individual cannot be neglected. We cannot in these days go back to the tenets of the school of Pythagoras in which all discoveries were attributed to the founder and which was probably only prevented from early decline by the oath that none of these were to be communicated to those not members. The evolution—for it is evolution and not revolution—which is taking place will need careful guidance. In astronomy, we in America need to make no apology for what has been done and what is being done. There is no group of scientific character that is so united, whose members are so willing to help each other and all who come for help. But we need to consider the future if the same standards of achievement are to be retained. The next generation will soon have to supply the places of those who now direct the work, and it is necessary to see that they are not only fully equipped, but that they are alive to changes in the methods of conducting research and ready to carry them out. Present achievement brings its own reward but the judgment of prosperity concerning ourselves will be largely based on the work of those who succeed us. We must pass on the torch not only lighted and burning brightly but well supplied with fuel. And for this help is needed from all. Those of you whose other occupations do not permit of the opportunity to engage actively in this work have done much and can do much by your active interest and, when necessary by material assistance. I believe that it is no accident that the Astronomical Society of the Pacific has so large a membership of non-professional astronomers. It records a real desire to know something more of our universe and to receive the stimulus which comes from a growing science. The more widespread such interest becomes, the more broadly will the science grow. The future holds much in store and I believe that as in the past you have sown freely and watered liberally so in the time to come, you will reap the harvest which is continually to be gathered in.